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Dynamical Creation of Entanglement and Steady Entanglement Between Two Spatially Separated Λ-Type Atoms

Guo Hui Yang · Xiang Yang Miao · Jun Feng Zhang

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Abstract We report possibility of generating entanglement and steady entanglement between two identical atoms in free space with a very natural way when their spatial separation is on the order of wavelength or less. We show a dynamical creation of entanglement and steady entanglement due to the radiative coupling with different separable initial atomic states and study the entanglement properties about this atomic subsystem. Not only the creation of steady state entanglement is decided by the initial atomic states, but also the magnitude of the entanglement and the steady state entanglement are found to be strongly dependent on the initial states. We derive a master equation for the atomic subspace and solve it analytically to show how the spontaneous emission from the two atoms system induces entanglement and steady entanglement, the crossing coupling terms in master equation can enhance the entanglement value.

Keywords Entanglement · Spontaneous emission · Atoms

1 Introduction

Dynamical creation of quantum entanglement has attracted a lot of attention due to their possible applications in quantum computing and quantum information [1, 2]. There have been many proposals for creating atomic entanglement [3–17], meanwhile some notable experimental demonstrations have also been performed. But in practical realization, every quantum system is open and unavoidable interaction with its environment which results in dissipation and destruction of entanglement.

Spontaneous emission in two-atom systems is an example of such noise which can destroy the entanglement [18]. However, as it is well known, in order to ensure that the induced dynamics is fast as compared to decoherence processes, the dipole-dipole interaction must be strong and thus the distance between the atoms must be small. In this case there is a substantial probability that a photon emitted by one atom will be absorbed by the other,

G.H. Yang (✉) · X.Y. Miao · J.F. Zhang
School of Physics and Information Engineering, Shanxi Normal University, Linfen 041004, PR China
e-mail: yangguohui_1981_1981@126.com

and then the photon exchange process can induce entanglement between atoms which can partially overcome decoherence caused by spontaneous. As a result, the system can decay to a stationary state which can be entangled even if the initial state was separable [19]. Further, most of the works listed above focused on the atom-atom entanglement consider only two-level schemes. To the best of our knowledge, the multilevel system especially the three-level atomic systems have been shown to be more powerful than two-level systems in quantum informatics; e.g., quantum key distribution [20, 21]. In a system of coupled multilevel atoms having closely lying energy states and interaction with vacuum, quantum interference between different radiative transition can occur, resulting in coherences which can induce entanglement. In Ref. [22], the authors explore the possibilities of creating radiatively stable entangled states of two three-level dipole interacting atoms by means of laser biharmonic continuous driving or pulses. However, the entanglement in three-level atoms interacting with a continuum via the retarded dipole-dipole interaction [23–27] has not been discussed carefully. It can opens up new channels in bath assisted entanglement in a very natural way.

In our this paper, we extend this kind of studies to investigate two radiatively coupled three-level atoms in Λ configuration. This system are coupled to a common vacuum field. The distance between these two atoms are of the order of the relative transition wavelength. So in this model we not only take into account the spontaneous emission and possible photon exchange between them but also consider the interaction between atoms and the vacuum field. We show that the vacuum field combined dissipation common induce entanglement.

2 Model Dynamics and Entanglement Negativity

We consider two identical three-level atoms in Λ configuration. The atoms (say A and B) have two degenerate ground states $|\mu\rangle_m$, $|g\rangle_m$ ($m = A, B$) and the excited state $|e\rangle_m$ as is shown in Fig. 1. The distance between them are of the order of relative transition wavelength. Assume that both the atoms interact with the common vacuum field and the transition dipole moments of atom A are parallel to the transition dipole moments of atom B. We put the atom A at the origin of coordinate system and the position of the atom B is given by the vector $\vec{R} = \vec{x}_B - \vec{x}_A$ which makes an angle ϕ with the x axis and an angle θ with the z axis, $\vec{x}_A$ ($\vec{x}_B$) are expressed as the position vectors of the two atoms. In addition, we assume that the dipole moments $\vec{d}_1$ and $\vec{d}_2$ which are corresponding to the transitions $|e\rangle_m \to |\mu\rangle_m$ and $|e\rangle_m \to |g\rangle_m$ are given by

$$\vec{d}_1 = \hat{x}d, \qquad \vec{d}_1 = \hat{y}d. \tag{1}$$

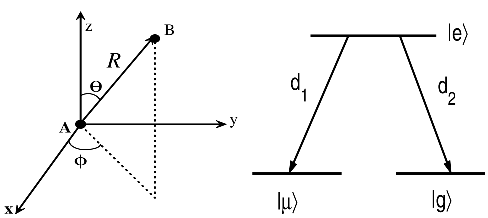

Fig. 1 The system of interest is comprised of two identical atoms in Λ configuration, which are located at $\vec{x}_A$ and $\vec{x}_B$, respectively. $\vec{R} = \vec{x}_B - \vec{x}_A$ is the relative position of atom B with respect to atom A. Level structure are shown in *right figure* of the atoms. d_1 and d_2 are the transition dipole matrix elements

Due to the spontaneous emission from the excited levels to the ground states and the distance between the two atoms is of the order of radiation wavelength, as is shown by Agarwal and Patnaik, in such atomic system there is possible the radiative process from one atom in the excited state loses its excitation which in turn excites the other atom from ground state to the excited state.

We adapt the standard derivation of a master equation to our atomic subsystem by using the Zwanzig projection operator method [28–31]. For this, we assume that the radiation field is initially in the vacuum state $\rho_F(0)$ and suppose that the total initial density operator is a product state $\rho(0)\rho_F(0)$, here $\rho(0)$ denotes the initial atomic state. By employing the Born approximation and Markoff approximation, tracing over the field states and using the rotating wave approximation to drop the antiresonant terms. One can obtain the master equation for the reduced atomic subsystem density operator takes the form

$$\dot{\rho} = -i[\Delta_1 + \Delta_2, \rho] + (L_I + L_{II} + L_{III})\rho, \tag{2}$$

where the symbols Δ_1 and Δ_2 are written as

$$\begin{aligned} \Delta_1 &= \Omega_1 \sigma^A_{e\mu} \otimes \sigma^B_{\mu e} + \Omega_2 \sigma^A_{eg} \otimes \sigma^B_{ge} + H.c. \\ \Delta_2 &= \Omega_{vc}[\sigma^A_{e\mu} \otimes \sigma^B_{ge} + \sigma^A_{eg} \otimes \sigma^B_{\mu e} + H.c.]. \end{aligned} \tag{3}$$

Here the terms L_α ($\alpha = I, II, III$) operators represent damping terms obtained as

$$\begin{aligned} L_I\rho &= \gamma_1[(2\sigma^A_{\mu e}\rho\sigma^A_{e\mu} - \sigma^A_{ee}\rho - \rho\sigma^A_{ee}) + A \to B] \\ &\quad + \gamma_2[(2\sigma^A_{ge}\rho\sigma^A_{eg} - \sigma^A_{ee}\rho - \rho\sigma^A_{ee}) + A \to B], \\ L_{II}\rho &= \Gamma_1[(2\sigma^B_{\mu e}\rho\sigma^A_{e\mu} - \sigma^A_{e\mu}\sigma^B_{\mu e}\rho - \rho\sigma^A_{e\mu}\sigma^B_{\mu e}) + h.c.] \\ &\quad + \Gamma_2[(2\sigma^B_{ge}\rho\sigma^A_{eg} - \sigma^A_{eg}\sigma^B_{ge}\rho - \rho\sigma^A_{eg}\sigma^B_{ge}) + h.c.], \\ L_{III}\rho &= \Gamma_{vc}[2\sigma^B_{\mu e}\rho\sigma^A_{eg} - \sigma^A_{eg}\sigma^B_{\mu e}\rho - \rho\sigma^A_{eg}\sigma^B_{\mu e} + h.c. \\ &\quad + 2\sigma^B_{ge}\rho\sigma^A_{e\mu} - \sigma^A_{e\mu}\sigma^B_{ge}\rho - \rho\sigma^A_{e\mu}\sigma^B_{ge} + h.c.]. \end{aligned} \tag{4}$$

In the above equations, the transition operator $\sigma^m_{e\mu} = |e\rangle_m\langle\mu|$ and $\sigma^m_{eg} = |e\rangle_m\langle g|$ are the dipole raising and lowering operators. From the process of deriving the above master equation, we have set $\omega_1 \approx \omega_2 = \omega_0$ since the ground state $|\mu\rangle$ and $|g\rangle$ are degenerate or nearly degenerate, here, ω_1 and ω_2 are the atomic frequencies corresponding to $|e\rangle_m \leftrightarrow |\mu\rangle_m$ and $|e\rangle_m \leftrightarrow |g\rangle_m$ transitions, respectively. As same to the calculations of Refs. [23–26], the coefficients appeared in the master equation can be written as

$$\begin{aligned} &\gamma_i = \gamma = \frac{2|\vec{d}|^2 k_0^3}{3}, \\ &\Omega_1 = \frac{3\gamma}{2}(P_r - \sin^2\theta\cos^2\phi Q_r), \qquad \Gamma_1 = \frac{3\gamma}{2}(P_i - \sin^2\theta\cos^2\phi Q_i), \\ &\Omega_2 = \frac{3\gamma}{2}(P_r - \sin^2\theta\sin^2\phi Q_r), \qquad \Gamma_2 = \frac{3\gamma}{2}(P_i - \sin^2\theta\sin^2\phi Q_i), \\ &\Omega_{vc} = -\frac{3\gamma}{2}\sin^2\theta\sin\phi\cos\phi Q_r, \qquad \Gamma_{vc} = -\frac{3\gamma}{2}\sin^2\theta\sin\phi\cos\phi Q_i \end{aligned} \tag{5}$$

where for $k_0 = \omega_0/c$ (c denotes the speed of light).

$$
\begin{aligned}
P_r &= \frac{\cos\xi}{\xi} - \frac{\sin\xi}{\xi^2} - \frac{\cos\xi}{\xi^3},\\
Q_r &= \frac{\cos\xi}{\xi} - \frac{3\sin\xi}{\xi^2} - \frac{3\cos\xi}{\xi^3},\\
P_i &= \frac{\sin\xi}{\xi} + \frac{\cos\xi}{\xi^2} - \frac{\sin\xi}{\xi^3},\\
Q_i &= \frac{\sin\xi}{\xi} + \frac{3\cos\xi}{\xi^2} - \frac{3\sin\xi}{\xi^3}
\end{aligned}
\tag{6}
$$

here, $\xi = k_0 R$, the γ_i $(i = 1, 2)$ terms describe the single atom spontaneous decay rate from the excited state $|e\rangle$ to the state $|\mu\rangle$ and $|g\rangle$. Γ_i and Ω_i terms represent the dipole-dipole coupling that are related to the decay and level shift of the collective atomic states, which couples a pair of parallel dipoles [30]. The new coherence terms Γ_{vc} and g_{vc} are cross coupling coefficients, which couple a pair of orthogonal dipoles, and it is strongly depend on the relative orientation of the atoms. From the above coefficients formulas it follows that the coupling coefficients are very small for large distance between the atoms and tend to be zero for $R \to \infty$. On the other hand, for $R \to 0$, Ω_1, Ω_2 and Ω_{vc} are very large and diverge in the unit of γ, whereas $\Gamma_1, \Gamma_2 \to \gamma$ and $\Omega_{vc} \to 0$ $(\theta \neq n\pi, \phi \neq n\pi/2)$. In the follow discussion about the entanglement properties, we will consider two special configurations of atomic system.

Configuration I: $\theta = n\pi$ i.e. both atoms lie along the z axis. In this case

$$\Gamma_1 = \Gamma_2 = \Gamma, \qquad \Omega_1 = \Omega_2 = \Omega$$

and the new coherence cross coupling terms $\Gamma_{vc} = \Omega_{vc} = 0$.

Configuration II: $\theta = \pi/2$ i.e. both atoms lie on the xy plane and $\phi = \pi/4$. In this case

$$\Gamma_1 = \Gamma_2 = \Gamma, \qquad \Omega_1 = \Omega_2 = \Omega$$

and the new coherence terms $\Gamma_{vc} \neq 0$, $\Omega_{vc} \neq 0$.

With knowing the master equation and the behavior of those coefficients, in order to describe the process of creation of entanglement between the two atoms, another point we must mention is the effective measure of mixed-state entanglement. For this atomic system, we mark the nine basis state $|ee\rangle$, $|e\mu\rangle$, $|eg\rangle$, $|\mu e\rangle$, $|\mu\mu\rangle$, $|\mu g\rangle$, $|ge\rangle$, $|g\mu\rangle$, $|gg\rangle$ as $1, 2, \ldots, 9$, here, $|jk\rangle = |j\rangle_A |k\rangle_B$ $(j, k = e, \mu, g)$. Under these basis states, the time evolution density matrix $\rho(t)$ can be written as

$$
\rho(t) = \begin{pmatrix}
\rho_{11(t)} & \cdots & \rho_{15(t)} & \cdots & \rho_{19(t)}\\
\vdots & & \vdots & & \vdots\\
\rho_{51(t)} & \cdots & \rho_{55(t)} & \cdots & \rho_{59(t)}\\
\vdots & & \vdots & & \vdots\\
\rho_{91(t)} & \cdots & \rho_{95(t)} & \cdots & \rho_{99(t)}
\end{pmatrix}.
\tag{7}
$$

For such atomic system we take a computable measure of entanglement proposed in [32], which is defined as

$$N(\rho)=\frac{\|\rho^{T_A}\|-1}{2}, \tag{8}$$

it is named negativity. This measure is based on the trace norm of the partial transposition $\|\rho^{T_A}\|$ of the state $\rho(t)$ [36]. From the Peres-Horodecki criterion of separability [33, 34], it notes that if ρ^{T_A} is not positive, then the state $\rho(t)$ is entangled. The negativity is an entanglement monotone and equivalent to the absolute value of the sum of the negative eigenvalues of ρ^{T_A}, i.e. $N(\rho)=-\sum_i \lambda_i$, where λ_i are the negative eigenvalues of ρ^{T_A}.

3 Entanglement and Steady Entanglement of This Atomic System

In this section, we study the process of creation of transient entanglement and steady state entanglement between atoms prepared in separable initial states with knowing the master equation and the behavior those coefficients. From the master equation, it notes us that there is no entanglement generated if both the atoms are all in the initial ground state $|\mu\rangle$ or $|g\rangle$, which means that the entanglement of this atomic system is zero forever for the initial states $|gg\rangle$, $|\mu\mu\rangle$, $|\mu g\rangle$ and $|g\mu\rangle$. So, if one want to obtain entanglement, the initial state of the atoms must satisfy the condition: at least one of them in the excited state $|e\rangle$ (or in it with some probability). On the other hand, the atoms we considered are Λ configuration, which contains two degenerate or nearly degenerate levels, so the entanglement evolution with this atomic system for the initial state $|e\mu\rangle$ should be similar to the case of initial state $|eg\rangle$. For simplicity, in the following we will consider two kind of separable initial states $|ee\rangle$ and $|e\mu\rangle$ to analytic the entanglement properties about our atomic subsystem.

We first consider the system is initially prepared in the pure state $|ee\rangle$ (both atoms are in the same excited state) and the cross coupling is absent (*configuration I*), for this case through checking the evolution density matrix at time t, we obtain the nonzero matrix elements of the evolution state $\rho(t)$ obey the following differential equations

$$\begin{aligned}
\dot\rho_{11} &= -8\gamma t\\
\dot\rho_{22} &= -4\gamma\rho_{22} - ig(\rho_{42}-\rho_{24}) - \Gamma(\rho_{42}+\rho_{24})\\
\dot\rho_{24} &= -4\gamma\rho_{24} - ig(\rho_{44}-\rho_{22}) - \Gamma(\rho_{44}+\rho_{22})\\
\dot\rho_{33} &= -4\gamma\rho_{33} - ig(\rho_{73}-\rho_{37}) - \Gamma(\rho_{73}+\rho_{37})\\
\dot\rho_{37} &= -4\gamma\rho_{37} - ig(\rho_{77}-\rho_{33}) - \Gamma(\rho_{33}+\rho_{77})\\
\dot\rho_{44} &= -4\gamma\rho_{44} - ig(\rho_{24}-\rho_{42}) - \Gamma(\rho_{24}+\rho_{42})\\
\dot\rho_{55} &= 2\gamma(\rho_{22}+\rho_{44}) + 2\Gamma(\rho_{24}+\rho_{42})\\
\dot\rho_{66} &= 2\gamma(\rho_{33}+\rho_{44}), \qquad \dot\rho_{88} = 2\gamma(\rho_{22}+\rho_{77})\\
\dot\rho_{68} &= 2\Gamma(\rho_{42}+\rho_{37})\\
\dot\rho_{77} &= -4\gamma\rho_{77} - ig(\rho_{37}-\rho_{73}) - \Gamma(\rho_{37}+\rho_{73})\\
\dot\rho_{99} &= 2\gamma(\rho_{33}+\rho_{77}) + 2\Gamma(\rho_{37}+\rho_{73}).
\end{aligned} \tag{9}$$

The differential equation $\dot\rho_{42}$ ($\dot\rho_{73}$, $\dot\rho_{86}$) is the conjugate $\dot\rho_{24}$ ($\dot\rho_{37}$, $\dot\rho_{68}$). Except these density matrix elements, others are zero. We can also compute the explicitly value of these matrix elements by using the Laplace transform method, they are following

$$\begin{aligned}
&\rho_{11}(t)=e^{-8\gamma t},\\
&\rho_{22}(t)=\rho_{44}(t)=\frac{2\gamma^2+\Gamma^2}{4\gamma^2-\Gamma^2}e^{-4\gamma t}\left[\cosh(2\Gamma t)-\frac{3\Gamma\gamma}{2\gamma^2+\Gamma^2}\sinh(2\Gamma t)-e^{-4\gamma t}\right],\\
&\rho_{24}(t)=\rho_{42}(t)=\frac{(3\Gamma\gamma)}{4\gamma^2-\Gamma^2}e^{-4\gamma t}\left[\cosh(2\Gamma t)-\frac{2\gamma^2+\Gamma^2}{3\Gamma\gamma}\sinh(2\Gamma t)-e^{-4\gamma t}\right],\\
&\rho_{55}(t)=\rho_{99}(t)=\frac{2e^{-4\gamma t}}{4\gamma^2-\Gamma^2}\big[\Gamma^2e^{-4\gamma t}+\gamma^2\cosh(4\gamma t)+2\Gamma\gamma\sinh(2\Gamma t)\\
&\qquad-(\Gamma^2+\gamma^2)\cosh(2\Gamma t)\big],\\
&\rho_{66}(t)=\rho_{88}(t)=\frac{e^{-4\gamma t}}{4\gamma^2-\Gamma^2}\big[2\gamma^2\cosh(4\gamma t)-\Gamma^2\sinh(4\gamma t)-2\gamma^2\cosh(2\Gamma t)\\
&\qquad+2\Gamma\gamma\sinh(2\Gamma t)\big],\\
&\rho_{68}(t)=\rho_{86}(t)=\frac{\Gamma e^{-4\gamma t}}{4\gamma^2-\Gamma^2}\big[2\Gamma\sinh(2\Gamma t)-2\gamma\cosh(2\Gamma t)+\Gamma e^{-4\gamma t}+\Gamma\cosh(4\gamma t)\big],\\
&\rho_{33}(t)=\rho_{77}(t)=\rho_{22}(t),\\
&\rho_{37}(t)=\rho_{73}(t)=\rho_{24}(t),
\end{aligned}\tag{10}$$

then we can calculate the entanglement about this system. From the definition of the negativity, the negative eigenvalues of ρ^{T_A} meet the equation

$$\begin{aligned}
&(\rho_{11}-\lambda)(\rho_{55}-\lambda)(\rho_{99}-\lambda)-\rho_{24}^2(\rho_{99}-\lambda)-\rho_{37}^2(\rho_{55}-\lambda)\\
&\quad-\rho_{68}^2(\rho_{11}-\lambda)+2\rho_{24}\rho_{37}\rho_{68}=0.
\end{aligned}\tag{11}$$

We note that the parameter Ω is not exist in the exact values of the density matrix elements, it cannot influence the entanglement of this system. In Fig. 2, we plot the evolution of negativity of the initial state $|ee\rangle$ with the cross coupling is absent. From the figure, we know that the maximal value of entanglement is very small (it is about 1.3×10^{-7}) and nearly to be zero for this case even when the distance between the two atoms is very closely. It also shows that there is no entanglement at earlier times, but with the evolute of time t, the

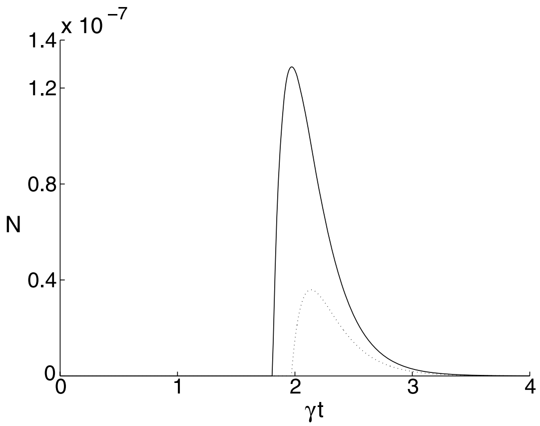

Fig. 2 The time evolution of negativity for initial state $|ee\rangle$ with the cross coupling is absent. The *solid line* and the *dotted line* are corresponding to $R=\lambda/10$ and $R=\lambda/8$, respectively

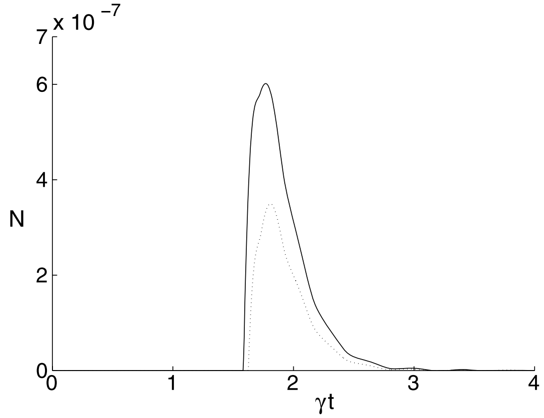

Fig. 3 The time evolution of negativity for initial state $|ee\rangle$ with the cross coupling $\Gamma_{vc} \neq 0$ and $\Omega_{vc} \neq 0$. *The solid line* and *the dotted line* are corresponding to $R = \lambda/4$ and $R = \lambda/3$, respectively

entanglement starts to build up. This is the example of phenomenon of delayed sudden birth of entanglement. The earlier unentangled process is corresponding to the process of spontaneous emission (atom A and atom B) and no photon exchange between atom A and B occurs. From the disentanglement process, it is clearly that there is no steady entanglement value, with the time t is increased, the spontaneous emission destroy every thing and make the entanglement disappeared finally.

To get insight into the effects about the crossing coupling terms on the entanglement of this atomic system. In Fig. 3, we give a detailed numerical analysis about the time evolution of negativity for the initial state $|ee\rangle$ with the cross coupling $\Gamma_{vc} \neq 0$ and $\Omega_{vc} \neq 0$ (*configuration II*). We clearly see that the steady entanglement is still disappeared, one cannot obtain the steady entanglement for this initial pure state whatever the cross coupling is absent or present. Comparing this figure to Fig. 2, it also notes us that the entanglement maximal value is still very small (it is about 6×10^{-7}). But the introducing of the crossing coupling terms can evidently enhance the entanglement value about this system even the distance between the two atoms is larger than the case of Fig. 2.

When the system is prepared in the initial pure state $|e\mu\rangle$ (atom A in the excited state and atom B in the ground state) and both atoms lie along the z axis (*configuration I*). Solving the master equation (the analytic method is similar to the case of initial state $|ee\rangle$), one can obtain all the nonzero matrix elements as following

$$\rho_{\alpha\alpha} = \frac{1}{2}e^{-4\gamma t}\big[\cosh(2\Gamma t) \pm \cos(2\Omega t)\big]$$

$$\rho_{24} = \rho_{42}^{*} = \frac{1}{2}e^{-4\gamma t}\big[i\sin(2\Omega t) - \sinh(2\Gamma t)\big]$$

$$\rho_{55} = \frac{\Gamma^2 - 2\gamma^2}{\sigma} + \frac{e^{-4\gamma t}}{\sigma}\big[(2\gamma^2 - \Gamma^2)\cosh(2\Gamma t) - \Gamma\gamma\sinh(2\Gamma t)\big]$$

$$\rho_{66} = \frac{\gamma^2(\delta + \sigma)}{\delta\sigma} + \gamma e^{-4\gamma t}\left[\frac{\gamma}{\sigma}\cosh(2\Gamma t) + \frac{\gamma}{\delta}\cos(2\Omega t) + \frac{\Gamma}{2\sigma}\sinh(2\Gamma t) - \frac{\Omega}{2\delta}\sin(2\Omega t)\right],$$

$$\rho_{68} = \rho_{86}^{*} = \frac{\Gamma^2}{2\sigma} - \frac{i\Omega\Gamma}{2\delta} + \Gamma e^{-4\gamma t}\left[\frac{i\Omega}{2\delta}\cos(2\Omega t) + \frac{i\gamma}{\delta}\sin(2\Omega t) - \frac{\Gamma}{2\sigma}\cosh(2\Gamma t) - \frac{\gamma}{\sigma}\sinh(2\Gamma t)\right].$$

$$\rho_{88} = \frac{\gamma^2(\delta-\sigma)}{\delta\sigma} + \gamma e^{-4\gamma t}\left[\frac{\gamma}{\sigma}\cosh(2\Gamma t) - \frac{\gamma}{\delta}\cos(2\Omega t) + \frac{\Gamma}{2\sigma}\sinh(2\Gamma t) + \frac{\Omega}{2\delta}\sin(2\Omega t)\right],$$

$$\rho_{37} = \rho_{73} = \rho_{33} = \rho_{77} = \rho_{99} = 0, \tag{12}$$

where $\alpha = 2, 4$ and $+(-)$ corresponding to $\rho_{33}(\rho_{77})$, the parameters $\sigma = \Gamma^2 - 4\gamma^2$ and $\delta = 4\gamma^2 + \Omega^2$. Then the entanglement negativity of the evolution states can be computed from the definition of it, the entanglement of this atomic system can be written as

$$N = \frac{1}{2}\left[\sqrt{\rho_{55}^2 + 4(|\rho_{24}|^2 + 4|\rho_{68}|^2)} - \rho_{55}\right]. \tag{13}$$

From the above formula, it notes us that the entanglement production is decided by the coherences ρ_{24} and ρ_{68}. If only one of them (or its conjugate) is nonzero, the system can be entangled. In Fig. 4, we plot the time evolution of entanglement negativity of initial state $|e\mu\rangle$ with different distance between the two atoms. As we see, the value of negativity is dependent on the distance significantly. When the distance is small, one can obtain a better value of N (for $R = \lambda/20$, it is about 0.46), and it is oscillating with time t. Another, the maximal value of entanglement for this initial state is evidently larger than the initial state $|ee\rangle$, the entanglement value is strongly dependent on the initial state. From the solutions (12) and (13), we found that only the coupling coefficient Ω is contained in the sine and cosine functions, which means that the oscillating periods are only decided by the constant Ω, it is different the case of initial state $|ee\rangle$ there is no oscillating (no sine and cosine functions is included in solutions (10)). However, with increasing the distance of R, this oscillating phenomenon is disappeared, actually it is existent, the reason we cannot see it is that the value of Ω becomes smaller with the increase of R, and oscillating becomes very weakly. On the other hand, from the disentanglement process, we can see clearly that the negativity is tend to be a finite value, which is evidently different the case of initial state $|ee\rangle$ and the case of V configuration atomic system [35], instead of decreasing to be zero the entanglement of our system is nonzero and tend to be a steady value, the steady state entanglement is appeared, it imply us that we can obtain the steady entanglement for the initial state $|e\mu\rangle$ or $|eg\rangle$, while the initial state $|ee\rangle$ can not do this. So for our atomic system, the steady state

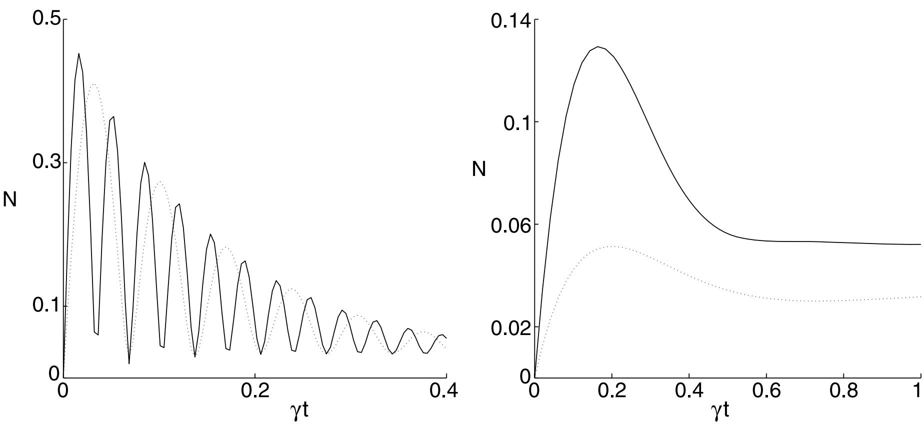

Fig. 4 The time evolution of negativity with different distance between the two atoms. For *the left figure*: *the solid line* and *the dotted line* corresponding to $R = \lambda/20$ and $R = \lambda/18$. For *the right figure*: *the solid line* and *the dotted line* corresponding to $R = \lambda/8$ and $R = \lambda/6$

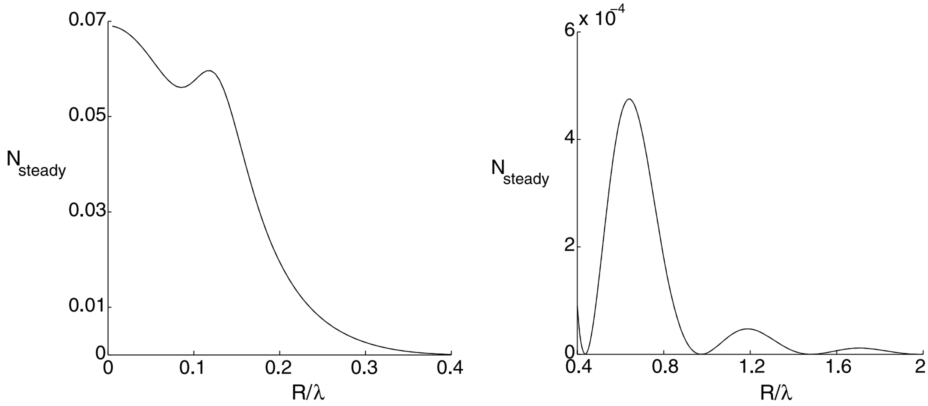

Fig. 5 The steady entanglement as a function of distance between the two atoms. We set $\gamma = 1$

entanglement is obtained with some appropriate initial states, and it is strongly dependent on the initial states and atomic configuration.

Now, we examine the steady state entanglement properties about the initial state $|e\mu\rangle$ with *configuration I*. From formula (12), which show us that in the limit of $t \to \infty$ there are some matrix elements are not zero. Those nonzero matrix elements are

$$\begin{aligned} \rho_{55} &= \frac{\Gamma^2 - 2\gamma^2}{\sigma}, \qquad \rho_{66} = \frac{\gamma^2(\delta+\sigma)}{\delta\sigma}, \\ \rho_{68} &= \rho_{86}^* = \frac{\Gamma^2}{2\sigma} - \frac{i\Omega\Gamma}{2\delta}, \qquad \rho_{88} = \frac{\gamma^2(\delta-\sigma)}{\delta\sigma}. \end{aligned} \tag{14}$$

Knowing these matrix elements, the steady entanglement N_{steady} is

$$N_{\text{steady}} = \frac{\sqrt{\rho_{55}^2 + 4|\rho_{68}|^2} - \rho_{55}}{2}, \tag{15}$$

we note that the symmetrical matrix elements ρ_{66} and ρ_{88} are not effect the entanglement of our system. The steady entanglement is only decided by the coherence term ρ_{68}. Since the value of $|\rho_{68}|$ is not zero, the system becomes entangled. In Fig. 5, it shows the steady state entanglement as a function of distance between the two atoms. In order to see clearly the steady entanglement properties we divide the distance R for two regions: from $0 \to 0.4\lambda$ and $0.4\lambda \to 2\lambda$. From the figure, we note that as we increase the distance between the two atoms, the degree of steady entanglement is oscillating weakly and progressively decreased. When the distance is enough large, there is no dipole-dipole interaction and the coherence term ρ_{68} is disappeared, the system is unentangled.

Then we consider the case of *configuration II* with initial state $|e\mu\rangle$, the cross coupling terms Γ_{vc} and Ω_{vc} are included. For this case, it is hard to give the analytical solutions about the master equation. So, here we solve numerically the master equation with different distance between the atoms and give out the entanglement properties. In Fig. 6, it plots the evolution of negativity with different distance R. As we see, the cross coupling between the atoms enhances the production of entanglement by comparing this figure with the right figure in Fig. 4. The reason is that the additional coupling between orthogonal dipoles produces some new coherences terms. Obviously, with the distance R is decreased, the maximal value

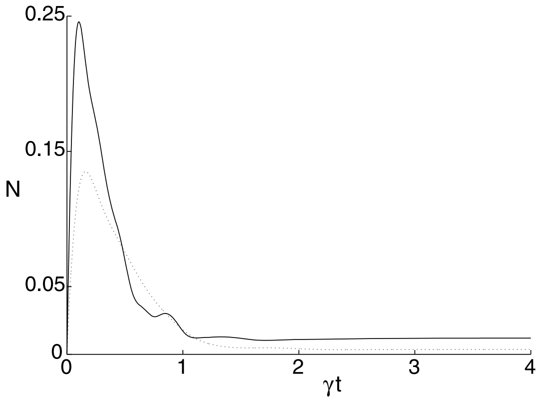

Fig. 6 The time evolution of negativity for initial state $|e\mu\rangle$ with the case of $\Gamma_{vc} \neq 0$ and $\Omega_{vc} \neq 0$. *The solid line* and *the dotted line* are corresponding to $R = \lambda/8$ and $R = \lambda/6$, respectively

of negativity is improved. As in the previous case, the steady entanglement is still existent. So, for our atomic system, we can obtain a steady entanglement in a very natural way (spontaneous emission). It not only can open up new channels in bath assisted entanglement and steady entanglement but also can give more control parameters. Which is different the case of [22], the authors explore the possibilities of creating radiatively stable entangled states of two three-level dipole interacting atoms by introducing the laser biharmonic continuous driving or pulses.

4 Conclusion

In conclusion, it is analyzed the entanglement properties of a pair of three-level atoms in the Λ configuration with vacuum induced coherences. We take into account the spontaneous emission and possible photon exchange between these two atoms when the distance is of the order of radiation wavelength. With different separable initial states, through studying the dynamical creation of entanglement and stable entanglement due to the collective effects which is present in the system, we show that the evolution of the entanglement and stable entanglement occurs due to radiative coupling between the two atoms via retarded dipole-dipole interaction. One can obtain a steady and larger entanglement for the initial states $|e\mu\rangle$ or $|eg\rangle$, while the excited states can not do this even if the two atoms are very closely. The creation of steady state entanglement is decided by the initial atomic states, and the magnitude of the entanglement and the steady state entanglement are found to be strongly dependent on the initial states. It also notes us that the introducing of the crossing coupling terms can enhance the entanglement about this atomic system if the two atoms lie on the xy plane. This system could opens up new channels in bath assisted entanglement in a very natural way and even could give more control parameters.

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